

Multivariate Countermonotonicity and the Minimal Copulas

Abstract:

As the maximum and minimum of bivariate copulas (in concordance order), Fréchet-Hoeffding upper and lower bounds play an important role in various bivariate optimization problems. Similarly, as the maximum of multivariate copulas, the Fréchet-Hoeffding upper bound is useful in multivariate optimization problems. However, there is no minimum copula available for dimensions $d \geq 3$. When the minimum copula is absent, minimal copulas can be useful for multivariate optimization problems. In this paper, we show that d -countermonotonic copulas defined in Lee and Ahn (2014) are minimal copulas and illustrate their usefulness in the variance minimization problem of the sum of random variables where the marginal distributions are not jointly mixable.